\* In the last blog, we have seen various tests MP Test, UMP Test, NP Lemma, Minimax Tests and Unbiased Test & Unbiased Critical Region.

But it is not enough to understand the tests just by definition. So now we will see the general formats and some examples.

*Coming to the general formats & examples*…..

* ***Most Powerful Test*** :- When null hypothesis is simple and alternative hypothesis is also simple then we use *Most Powerful Test* (MP Test).

The region C is called MP Critical Region of size α for testing

H0 : Ɵ = Ɵ0 vs H1 : Ɵ = Ɵ1

**Step I** :- Find the likelihood function of Ɵ under H0 and H1.  L0 and L1

**Step II** :- Then by using Neyman-Pearson Lemma (NP Lemma) find the best critical region (BCR).

L1/ L0≥ K, for all K > 0

Where,

L1 = Likelihood function under H1

L0 = Likelihood function under H0

K = Constant

∴ Best Critical Region is

C = { :L1/ L0≥ K }

Now, have a look on the example.

**Ex** :- A random sample of size n is taken from Poisson Distribution with parameter λ. Construct a MP test of levelα for testing H0 : λ = λ0 against H1 : λ = λ1 (>λ0).

**Sol** :-

*Aim* – To construct MP test

Let 1, 2, ….. , n is a random sample from Poisson distribution

X ~ P(λ)

PMF : f() = ; = 0,1,2,…..

λ > 0

= 0 ; otherwise

The likelihood function is given by,

L = L(λ|) =

L =

To test H0 : λ = λ0 Vs H1 : λ = λ1 > λ0

∴ L1 =

L0 =

The MP test can be constructed by using NP Lemma

≥ K

≥ K

(-nλ1) + logλ1 ≥ K(-nλ0) + logλ0

n(λ1 -λ0) + log ≥ K

≥

≥ K’ ()

MP test of level α is

Reject H0 if ≥ K’

Where K’ is positive constant which can be obtained by following equation

P( ≥ K’ | λ = λ0) = α

So, we have seen the general criteria and example of Most Powerful (MP) Test. Now we will come towards Uniformly Most Powerful (UMP) Test.

* ***Uniformly Most Powerful Test*** :- When null hypothesis is simple and alternative hypothesis is not simple then we use *Uniformly Most Powerful Test* (UMP Test).

The region C is called UMP Critical Region of size α for testing

H0 : Ɵ = Ɵ0 vs H1 : Ɵ ≠ Ɵ1

**Step I** :- Find the likelihood function of Ɵ under H0 and H1.  L0 and L1

**Step II** :- Then by using Neyman-Pearson Lemma (NP Lemma) find the best critical region (BCR).

L1/ L0≥ K, for all K > 0

Where,

L1 = Likelihood function under H1

L0 = Likelihood function under H0

K = Constant

∴ Best Critical Region is

C = { :L1/ L0≥ K }

Now, have a look on the example. For better understanding let’s again take the example of Poisson Distribution.

**Ex** :- A random sample of size 20 is drawn from Poisson distribution with mean λ derive an UMP test for H0 : λ = 1/10 against H1 : λ > 1/10 at α = 0.05

**Sol** :-

*Aim* :- To construct UMP test

For testing H0 : λ = 1/10 against H1 : λ > 1/10 at α = 0.05

Let 1, 2, ….. , 20 is a random sample from Poisson distribution

X ~ P(λ)

PMF : f() = ; = 0,1,2,…..

λ > 0

= 0 ; otherwise

Using NP Lemma,

≥ K

Likelihood function is given by,

L = L(λ|) =

L =

The UMP test can be constructed by using NP Lemma

≥ K

≥ K

(-nλ1) + logλ1 ≥ K(-nλ0) + logλ0

n(λ1 -λ0) + log ≥ K

≥

≥ K1

Best critical region is

C = { : > K1}

Where K1 : P[ > K1| H0] = α

We know that T = ~ Pois(n, λ0), under H0

∑P[T = t | H0] ≤ α

1 - ≤ α

1 - ≤ 0.05

Therefore K1 = 6

Best critical region is rewritten as

C = { : > 6}